Chapter 3

Gravitational Transformations of the Units of Physics

Covariance of the Laws of Physics: From the event horizon of a black hole to the most isolated volume in the universe, there are big differences in the rates of time throughout the universe. How do the laws of physics remain the same when the rate of time is different between locations? Why does a rate of time gradient also not affect the laws of physics? It is an oversimplification to imagine that changing the rate of time is similar to running a movie in slow motion while keeping the laws of physics unchanged.

We are going to be looking at how nature maintains the laws of physics when the rate of time changes with gravitational gamma. This is not just an academic question. Gravity produces a rate of time gradient and a gradient in the coordinate speed of light. Therefore, even in earth's gravity, the simple act of lifting an object to a different elevation means that the object is moved to a location where there is a different rate of time and a different coordinate speed of light. Acknowledging that there are changes in the rate of time leads to surprising new physical insights.

When the rate of time is different between two locations, but the laws of physics are the same, there must also be other changes in the units of physics to offset the difference in the rate of time. For example, momentum scales proportional to $1/t$, force scales proportional to $1/t^2$, power scales proportional to $1/t^3$ and the fine structure constant is independent of time ($1/t^0$). This is time raised to four different powers, yet the laws of physics are constant even with this difference in time dependence. What additional changes are required to offset the change in the rate of time and preserve the laws of physics unchanged in different gravitational potentials?

If there is a coordinate rate of time in a zero gravity location that is different from the rate of time in a location with gravity, and if the coordinate speed of light is different in the two locations, shouldn't there also be a difference in at least some of the other units of physics? For example, is one Joule of energy or one Newton of force also different in the zero gravity location compared to the gravity location? To make a meaningful comparison of the units of physics between locations with different gravitational potentials, it would be necessary to use a single rate of time. This point is easy to see. The more difficult question is: How do we treat length in this exercise?

It is impossible to directly compare length between two locations with a different gravitational potential. Also vectors are ambiguous when compared between locations with different gravitational potential. For example, the direction of a vector can be different depending on the path chosen to transport a vector between two locations with different gravitational potentials.

Therefore adopting the locally measured proper length as a standard of length for that location eliminates ambiguity, but is it the length standard we are seeking?

To be clear, this exercise is not interested in calculating the general relativistic effects on space and time. We will obtain this information from standard general relativity calculations. We will presume that we already know the gravitational gamma ($\Gamma = dt/d\tau$) for each location of interest. Instead we are interested in understanding how the laws of physics accommodate the spatial and temporal differences associated with these different values of Γ. The laws of physics always scale in a way that keeps the speed of light constant ($c = dL/d\tau$). For example, a zero gravity observer might perceive that that a location in gravity has a slow rate of time. However, the zero gravity observer also perceives that this location in gravity also has a proportionately slow coordinate speed of light. A speed of light experiment performed in gravity always results in the universal constant c because a zero gravity observer perceives a slow coordinate speed of light being timed by a slow clock. This results in not only a constant proper speed of light (c) but also the zero gravity observer can consider proper length as constant (independent of Γ). In other words, when the zero gravity observer applies his/her rate of time and adjusts for the different coordinate speed of light, then the unit of length (L) can be considered constant.

All the forces scale with proper length. This is true for not only the electromagnetic force, but even gravitational acceleration scales with proper length. In the last chapter we showed that $g = c^2 \Delta T / \Delta \hbar d\tau$. In this equation $\Delta \hbar$ is an increment of proper length in the elevation direction. General relativity tells us that there is a difference between circumferential radius R and proper length L. There is also one perspective where the tangential proper length decreases relative to coordinate tangential length when gravity is introduced into a volume of spacetime. However, fermions, bosons and forces know nothing about the general relativistic effects involving circumferential radius or coordinate tangential length. These particles and forces all scale with proper length. If gravity produces non Euclidian spatial geometry, these particles and forces merely accept the proper volume at a particular location and scale with proper length. Therefore since fundamental particles and forces scale with proper length and proper volume, for this exercise we need to adopt a coordinate system that recognizes proper length as a standard.

Normalized Coordinate System: The conclusion of this is that the analysis we wish to perform on the covariance of the laws of physics is best accomplished by adopting a coordinate system that uses coordinate rate of time from general relativity as the time standard and proper length as our length standard. This is an unconventional coordinate system that is a hybrid between the Schwarzschild coordinate system (coordinate time and circumferential radius) and the standard coordinates that use proper time and proper length. This coordinate system will be used to analyze the covariance of the laws of physics when two locations have different gravitational potentials and different rates of time. In this analysis we always assume both a constant distance between locations and static gravity. Further support for the use of this coordinate system will be offered by actually performing this analysis using this hybrid coordinate system and seeing if the results are reasonable.

The hybrid coordinate system that uses proper length and coordinate rate of time will be called the “normalized” coordinate system. The speed of light utilizing this coordinate system will be called the “normalized” speed of light. We will also be referring to the “normalized” unit of energy, force, etc. All of these units use proper length and zero gravity rate of time. The normalized coordinates cannot be used for general relativity calculations to determine spacetime curvature. The equations become so simplified that important information is lost. Instead, the normalized coordinate system accepts the value of Γ obtained from general relativity and utilizes this information to analyze other aspects of physics. By adopting proper length as our coordinate length and zero gravity rate of time we achieve a coordinate system that works well with quantum mechanics and gives insights into the forces of nature.

Length and Time Transformations: The following analysis will use dimensional analysis and therefore we will be using the symbols of dimensional analysis. These are: *M, L, T, Q*, and *Θ* to represent mass, length, time, charge and temperature respectively. For example, the units of energy are: kg m^2/s^2. The conversion to dimensional analysis terminology is:
kg m^2/s^2 → ML^2/T^2.
Also, the calculations to follow will be making transformations between the various units of physics when Γ changes. For example, to understand how the laws of physics are maintained going from a hypothetical location in zero gravity ($\Gamma = 1$) to a location with strong gravity ($\Gamma > 1$), we will be working with discrete units such as a Joule or a Newton. This means that the transformation of our coordinates also requires the use of discrete units of length and time rather than the differential form.

For example, $dt = \Gamma d\tau$ relates the rate of coordinate time (dt) to the rate of proper time in gravity ($d\tau$). In this case $dt > d\tau$. However, suppose we compare a unit of time, such as one second in a location with zero gravity to one second in a location with gravity. If each location sent out a light pulse lasting 1 second (according to a local clock), then any observer (independent of Γ) would agree that the light pulse from the location in gravity lasted longer than the light pulse from the zero gravity location. This will be represented as $T_g > T_o$. The subscript “*g*” represents a location in gravity and “*o*” represents a location with zero gravity. Therefore, $T_g > T_o$ represents that a unit of time in gravity is larger (longer) than a unit of time in zero gravity. When we convert $dt = \Gamma d\tau$ to express a relationship between units of time it becomes:

$T_o = T_g/\Gamma$ unit of time transformation from zero gravity to gravity

There is no new physics being expressed here. The difference is comparable to comparing a rate of pulses expressed as pulse per second compared to the time between pulses expressed as seconds per pulse.

Since proper length is adopted as our standard of length, this means that we are not making a distinction between proper length in any location or orientation. The way that this is expressed is:

$L_o = L_g$ unit of length transformation from zero gravity to gravity

Normalized Speed of Light: When the normalized coordinate system uses proper length and the zero gravity rate of time, then the normalized speed of light (designated with a capital "C") becomes: $C = dL/dt$. In other words, the normalized speed of light in the normalized coordinate system is the change in proper length divided by the rate of coordinate time dt.

$C \equiv dL/dt = cd\tau/dt = c/\Gamma$ C = normalized speed of light
$C_o = c$ C_o = normalized speed of light in zero gravity (when $\Gamma = 1$)
$C_g = c/\Gamma$ C_g = normalized speed of light in a location with gravity (when $\Gamma > 1$)
$C_o = \Gamma C_g$ relationship between C_o and C_g

If there are two locations (1 and 2) that have gravitational gammas Γ_1 and Γ_2 respectively, then they will have normalized speed of light of C_1 and C_2. The relation between these two different normalized speeds of light is: $\Gamma_1 C_1 = \Gamma_2 C_2 = c$.

In the equation $C_o = \Gamma C_g$ we have eliminated the need for Γ_1 because in zero gravity $\Gamma = 1$ and the need to mention Γ_1 disappears. It is informative to give an example of $C_o = \Gamma C_g$. The gravitational gamma Γ at the surface of the sun is: $\Gamma \approx 1.000002$. If we set the normalized speed of light in zero gravity to $C_o = 1$, then the surface of the sun has $C_g \approx 0.999998$. Since proper length is coordinate length in the normalized coordinate system, the non-Euclidian properties of space are interpreted as gravity creating additional proper volume in the space surrounding a mass. Therefore the non-Euclidian volume surrounding the sun is merely accepted by the normalized coordinate system. If a beam of light passes through this non Euclidian volume of space, then the difference in optical path length across the width of the beam is taken into account. This difference in path length contributes to bending of the light.

Comparison of Coordinate Speed of Light: We previously designated the coordinate speed of light using Schwarzschild coordinates as $\mathcal{C}_R$ and $\mathcal{C}_T$. The comparison of the normalized speed of light in gravity C_g to the Schwarzschild coordinate speed of light is:

Length Transformation	Coordinate Speed of Light	Speed of Light Conversion	Coordinates
$L_o = L_g$	$C_g = dL/dt$	$c = \Gamma C_g$	Normalized speed of light conversion
$dR = (1/\Gamma)dL_R$	$\mathcal{C}_R = dR/dt$	$c = \Gamma^2 \mathcal{C}_R$	Schwarzschild coordinates (radial)
$Rd\Omega = dL_T$	$\mathcal{C}_T = Rd\Omega/dt$	$c = \Gamma \mathcal{C}_T$	Schwarzschild coordinates (tangential)

The normalized speed of light is similar to the proper speed of light c in the sense that both are independent of orientation (radial or tangential). The only difference is that the normalized speed of light uses coordinate time which is an absolute standard for the rate of time and also is a faster rate of time compared to the proper rate of time in gravity (stationary frame of reference).

Internally Self Consistent: Another important similarity between the normalized speed of light and proper speed of light is:

$$dL = Cdt = c\,d\tau$$

The above relationship indicates that the normalized coordinate system is internally self consistent. By definition, the following is always true: $cd\tau = dL$ (any orientation or gravitational Γ). So also the following is always true: $Cdt = dL$ (any orientation or gravitational Γ).

In gravity, the normalized speed of light is slow $(C_g = c/\Gamma)$. However, a unit of time in gravity T_g is longer than the same unit of time in zero gravity $(T_g = \Gamma\ T_o)$. The combination of these two factors offset each other, thereby producing a constant length: $C_gT_g = (c/\Gamma)\ (\Gamma\ T_o) = L_o$. The combination of these two factors achieves the same length in any Γ or orientation. Therefore it is possible to say that $L_o = L_g$ and have a coordinate system that is internally consistent.

Energy Transformation: We know the transformation of units of length $(L_o = L_g)$ and time $(T_o = T_g/\Gamma)$, but we need to determine the transformation for units of mass before all other transformations can be easily calculated. It is not obvious what transformation mass would be if we used a standard unit of mass in zero gravity (M_o) to quantify the same proper unit of mass in gravity (M_g). Mass is not synonymous with matter. For example, one electron in zero gravity transforms into one electron in gravity. However, do they have the same inertia? Mass is a quantification of inertia which implies force and acceleration. Both of these involve time, so it should be expected that mass (inertia) may have some dependence on the rate of time. Since we cannot directly reason to the mass transformation, it is necessary to determine some other transformation between zero gravity and gravity that can be determined by physical reasoning. Then we will use that transformation to deduce the mass transformation indirectly. Fortunately, there are two additional transformations that can be determined by physical reasoning. The conservation of momentum implies that a unit of momentum in zero gravity equals a unit of momentum in gravity. This will be written as $p_o = p_g$. The second transformation that can be determined from physical reasoning involves energy. It will be shown that $E_o = \Gamma E_g$.

Before actually deriving this, I just want to review the meaning of $E_o = \Gamma E_g$. The term E_o represents a unit of energy (such as 1 Joule or 1 eV) in a location with zero gravity ($\Gamma = 1$). Similarly, E_g represents the same unit of energy in a location with gravity ($\Gamma > 1$). Furthermore, we assume that both sources of energy can be considered essentially stationary relative to each other. Therefore since gamma is greater than one ($\Gamma > 1$) the equation $E_o = \Gamma\ E_g$ says that 1 Joule in zero gravity represents more energy than 1 Joule in a location with gravity ($\Gamma > 1$). The ratio of these two energies is $E_o/E_g = \Gamma$.

The simplified energy transformation involves an approximation while the momentum transformation does not. However, we will start with the energy transformation anyway because it better illustrates the meaning of terms. To make a comparison of energy in different gravitational potentials, both E_o and E_g must be measured using the same standard of energy which implies using the same rate of time for both measurements. Perhaps it is convenient to imagine using the zero gravity (coordinate) rate of time for both energy measurements, but the only requirement is that the same rate of time be used. For example, it will be shown that an electron in gravity has less energy than an electron in zero gravity when the energies are compared using the same rate of time. Both electrons have 511,000 eV measured locally, but the energy standard (1 eV) changes. The proportionality constant is the gravitational gamma:

$$\Gamma = \frac{1}{\sqrt{\left(1 - \frac{2Gm}{c^2 R}\right)}} = \frac{dt}{d\tau}$$

This concept is best explained with a thought experiment. Suppose that there is a planet that is in a highly elliptical orbit around a star. The planet's kinetic energy changes from a minimum kinetic energy at the orbital apogee to a maximum kinetic energy at the perigee. Does this change in kinetic energy produce any change in the gravity produced by the combination of the planet and star as the planet orbits the star? (Assume the measurement is made far from the star/planet). We know from general relativity that the total gravity produced by a closed system remains constant when there is no transfer of energy into or out of the closed system. Since gravity scales with energy, the implication is that the planet's total energy remains constant in all parts of the highly elliptical orbit. It is obvious that the planet's kinetic energy changes from apogee to perigee. Since the gravity produced by the orbiting planet is constant, the implication is that the gain in kinetic energy equals the loss of internal energy as the planet enters a stronger gravitational field. The more general principle is that a body in free fall maintains a constant total energy (internal plus kinetic) as it falls. This can be expressed as: $E_o = E_g + E_k$ where:

E_o is the internal energy of a mass m in zero gravity ($E_o = m_o c^2$) measured using the zero gravity standard of energy.
E_g is the internal energy of a mass in gravity but measured using the zero gravity standard of energy (measured using the coordinate clock).
E_k is the kinetic energy of mass m in free fall from infinity to distance r in the gravity of larger mass M. Also E_k is measured using the zero gravity standard of energy.

Since the definition of E_k references zero gravity, then $E_k = Gm_oM/r = E_o(GM/c^2r)$. In the following calculation we will use the approximation: $E_o(GM/c^2r) \approx E_g(GM/c^2r)$. Normally this approximation would not be allowed since we are attempting to determine the relationship between E_o and E_g, however in this case it is acceptable because in weak gravity $(GM/c^2r) << 1$..

$E_o = E_g + E_k$ $\quad$ $E_k \approx E_g(GM/c^2r)$ approximation from above
$E_o \approx E_g(1 + GM/c^2r)$ $\quad$ set: $\Gamma \approx 1 + GM/c^2r$
$E_o \approx \Gamma E_g$ $\quad$ this approximation is exact in a rigorous analysis

The gravitational red/blue shift can cause some confusion in the discussion of standards of energy and will be discussed later. Also, this equation can easily be misinterpreted if proper units of energy are used to measure E_g rather than always using normalized units of energy.

Mass Transformation from Energy: Next we will solve for the mass transformation using $E_o = \Gamma E_g$; $L_o = L_g$ and $T_o = T_g/\Gamma$. Energy has units of kg m²/s² which in dimensional analysis terms will be expressed as: $E \rightarrow M L^2/T^2$

$$E_o = \Gamma E_g \qquad \text{set: } E_o \rightarrow \frac{M_o L_o^2}{T_o^2} \quad \text{and} \quad E_g \rightarrow \frac{M_g L_g^2}{T_g^2}$$

$$\frac{M_o L_o^2}{T_o^2} = \Gamma \left(\frac{M_g L_g^2}{T_g^2}\right) \qquad \text{set: } L_g = L_o \text{ and } T_g = \Gamma\ T_o$$

$$\frac{M_o L_o^2}{T_o^2} = \Gamma \left(\frac{M_g L_o^2}{\Gamma^2 T_o^2}\right)$$

$M_o = M_g/\Gamma$ $\quad$ units of mass transformation obtained from the energy transformation

Again, both M_o and M_g represent the same units of stationary mass such as 1 kilogram. Furthermore, both units of mass are measured using a single rate of time. As suspected previously, the connection between mass and inertia means that the normalized unit of mass (inertia) has a dependence on the rate of time (Γ dependence).

The transformation $M_o = M_g/\Gamma$ looks strange because it says that the normalized mass unit increases as gravity increases (as Γ increases). This will be analyzed later, but it relates to the inertia measured by a zero gravity observer.

Mass Transformation from Momentum: Next we will check the mass transformation by also deriving the mass transformation by assuming the conservation of momentum ($p_o = p_g$). This becomes:

$$\frac{M_o L_o}{T_o} = \frac{M_g L_g}{T_g} \qquad \text{set } L_o = L_g \text{ and } T_o = T_g/\Gamma$$

$M_o = M_g/\Gamma$

Now that we have established the mass transformation, we can combine this with the length and time transformations to generate all of the other transformations nature requires to maintain the laws of physics when the rate of time changes because of a change in gravitational potential. To summarize, here are the key transformations:

$L_o = L_g$	unit of length transformation
$T_o = T_g/\Gamma$	unit of time transformation
$M_o = M_g/\Gamma$	unit of mass transformation

Appendix B at the end of this chapter shows the details of derivation of the various transformations. Essentially this is just dimensional analysis where the dimensions of mass, length and time are transformed from zero gravity to gravity. The following transformation of force is typical of the other transformations.

Force F: $F_o \to \dfrac{M_o L_o}{T_o^2} = \dfrac{\left(\frac{M_g}{\Gamma}\right) L_g}{\frac{T_g^2}{\Gamma^2}} \to \Gamma F_g$

$F_o = \Gamma F_g$ force transformation

The table on the following page gives all of the important transformations.

Gravitational Transformation of Units and Constants: The following are transformations of units of physics from zero gravity $\Gamma = 1$ to a location in gravity $\Gamma > 1$. The relationships are expressed assuming a single rate of time and proper length. The gravitational gamma Γ is defined as: $\Gamma \equiv \frac{dt}{d\tau} = \frac{1}{\sqrt{1-\left(\frac{2Gm}{c^2R}\right)}} \approx 1 + Gm/c^2R$. The symbols of dimensional analysis L, T, M, Q and Θ are used to represent length, time, mass, charge and temperature respectively.

Normalized Transformations

$L_o = L_g$	unit of length transformation
$T_o = T_g/\Gamma$	unit of time transformation
$M_o = M_g/\Gamma$	unit of mass transformation
$Q_o = Q_g$	unit of charge expressed in coulombs – not stat coulombs
$\Theta_o = \Theta_g$	unit of temperature transformation
$C_o = \Gamma\, C_g$	normalized speed of light transformation
$dL = \Gamma\, dR$	proper length and circumferential radius transformation

$E_o = \Gamma E_g$	energy
$v_o = \Gamma v_g$	velocity
$F_o = \Gamma F_g$	force
$P_o = \Gamma^2 P_g$	power
$G_o = \Gamma^3 G_g$	gravitational constant
$U_o = \Gamma U_g$	energy density
$\mathbb{P}_o = \Gamma \mathbb{P}_g$	pressure
$\omega_o = \Gamma \omega_g$	frequency
$\rho_o = \rho_g/\Gamma$	density
$k_o = \Gamma\, k_g$	Boltzmann's constant
$\sigma_o = \Gamma^2\, \sigma_g$	Stefan-Boltzmann Constant
$\mathbb{I}_o = \Gamma \mathbb{I}_g$	electrical current
$\mathbb{V}_o = \Gamma \mathbb{V}_g$	voltage
$\varepsilon_{oo} = \varepsilon_{og}/\Gamma$	permittivity of vacuum
$\mu_{oo} = \mu_{og}/\Gamma$	permeability of vacuum

Units and Constants That Do Not Change in Gravity

$p_o = p_g$	momentum is conserved
$\mathcal{L}_o = \mathcal{L}_g$	angular momentum is conserved
$\hbar_o = \hbar_g$	Planck's constant (angular momentum is conserved)
$\alpha_o = \alpha_g$	fine structure constant (dimensionless constant is conserved)
$\Omega_o = \Omega_g$	electrical resistance
$\mathbb{B}_o = \mathbb{B}_g$	magnetic flux density
$Z_{oo} = Z_{og}$	impedance of free space
$Z_{so} = Z_{sg}$	impedance of spacetime

Fundamental Equations

$E_o = E_g + E_k$	relationship of internal energy and gravitational kinetic energy E_k
$E_o = E_g - E_{po}$	relationship of internal energy and gravitational potential energy E_{po}

Conversion to Normalized Perspective: When we calculate energy, velocity, force, mass, power, voltage, etc. using proper time, because of the covariance of the laws of physics, we obtain an answer that numerically equals the value in zero gravity. Therefore, to convert this proper value into normalized values, we merely substitute the proper value into the above transformations by replacing the zero gravity term (E_o, V_o, F_o, etc.) and solve for the normalized value (E_g, V_g, F_g, etc.). For example, an electron has energy of 8.187×10^{-14} Joules. In the normalized perspective, this is really the energy of an electron in zero gravity. However, the covariance of the laws of physics allows us to use this energy in locations with $\Gamma > 1$. To convert to normalized energy, we merely substitute the zero gravity energy (8.187×10^{-14} Joules) for E_o in the equation $E_g = E_o/\Gamma$ and solve for E_g. Since $\Gamma > 1$ for a location in gravity, this means that $E_g < E_o$. For example, at the surface of the sun $\Gamma \approx 1 + 2 \times 10^{-6}$. Therefore an electron at the surface of the sun has only 0.999998 the energy of an electron in zero gravity.

Insights from Transformations

Energy Transformation and Calculation: We normally closely associate mass and energy. However, the normalized transformations treat mass and energy differently. When an object is moved to a location with a larger gravitational gamma (and slower rate of time), the normalized energy of the object decreases and the normalized mass (inertia) of the object increases. The mass transformation is discussed below, but when we transform $E = mc^2$ into the normalized time perspective, the gravitational effect on the normalized speed of light is squared and the gravitational effect on mass is raised only to the first power. The result is: $E_o = \Gamma E_g$. This equation applies only to stationary objects because this was an assumption used in the derivation. The term "stationary" means no change as a function of time in the optical path length between two objects or points.

The equation $E_o = \Gamma E_g$ says that a unit of energy in zero gravity is larger than the same unit of energy in a location with gravity. This applies to all forms of energy such as: the annihilation energy of mass, the energy stored in capacitors, the energy of chemical reactions, thermal energy, or the energy of atomic transitions. In all cases the normalized energy of stationary objects in gravity is diminished by the gravitational gamma factor. The loss of energy when an object moves to stronger gravity is easy to see. A meteor striking the earth generates heat. This heat is the lost internal energy of the meteor. The atoms of the meteor that remain in the earth's gravity have less internal energy than they had in space (once the heat is removed).

If we elevate a one kilogram mass by 1 meter in earth's gravity, we say that we have given the mass potential energy of 9.8 Joules. Where is this energy stored? I want to see and understand this mysterious gravitational potential energy. If we ignore the change in time over the one meter elevation, then the source of gravitational potential energy is a mystery. However, if we acknowledge that the rate of time is different when we change elevation, then we arrive at the conclusion that the energy expressed in normalized units of energy is also different at the two elevations. This insight allows us to obtain a partial insight into the storage of gravitational potential energy. The proposed particle model presented later will give a more complete explanation. Below, we will use the transformation $E_o = \Gamma E_g$ and compare E_g at two different elevations (1 and 2 where elevation 2 is higher than 1). We will use weak gravity approximations and the following symbols:

Normalized energy in gravity at elevation 1 and 2:	E_{g1} and E_{g2},
Large mass (planet) and small test mass	M and m
Radius from the center of the large mass to elevation 1 and 2:	r_1 and r_2
Gravitational gamma and beta for elevation 1 and 2:	Γ_1, Γ_2, β_1, and β_2

$E_{g2} = \dfrac{\mathrm{E_o}}{\Gamma_2}$ and $E_{g1} = \dfrac{\mathrm{E_o}}{\Gamma_1}$ normalized energy transformations

$E_{g2} - E_{g1} = \dfrac{\mathrm{E_o}}{\Gamma_2} - \dfrac{\mathrm{E_o}}{\Gamma_1} = E_o\left(\frac{1}{\Gamma_2} - \frac{1}{\Gamma_1}\right) = E_o(\beta_1 - \beta_2)$ set: $\frac{1}{\Gamma} = 1 - \beta$

$E_{g2} - E_{g1} \approx \left[\left(\dfrac{GM}{c^2 r_1}\right) - \left(\dfrac{GM}{c^2 r_2}\right)\right] mc^2 \approx \left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)(GMm)$

since $r_2 - r_1 << r_1$ substitute: $\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) \approx \dfrac{r_2 - r_1}{r_1^2}$

$E_{g2} - E_{g1} \approx (r_2 - r_1)\left(\dfrac{GM}{r_1^2}\right)m$ set $r_2 - r_1 = \Delta\hbar$ and acceleration $\dfrac{GM}{r_1^2} = g$

$E_{g2} - E_{g1} \approx gm\,\Delta\hbar$ gravitational potential energy

Therefore, the change in the normalized energy between level 1 and 2 $(E_{g2} - E_{g1})$ equals the gravitational potential energy for mass m in gravitational acceleration g for a height change of $\Delta\hbar = r_2 - r_1$. It is interesting to do a numerical example using a one kilogram mass being elevated by one meter in the earth's gravity.

$M = 5.976 \times 10^{24}$ kg mass of the earth

$r_1 = 6.378 \times 10^6$ m radius of the earth

$\Gamma_1 \approx 1 + GM/c^2 r_1 = 1 + 6.977 \times 10^{-10}$ gravitational gamma of the earth at sea level

$\Gamma_2 \approx \Gamma_1 + (\Gamma_1 - 1)[(r_2 - r_1)/r_1] \approx 1 + 6.977 \times 10^{-10} + 1.091 \times 10^{-16}$

$\Gamma_2 - \Gamma_1 \approx 1.091 \times 10^{-16}$ difference in Γ when $\Delta\hbar = r_2 - r_1 = 1$ meter

$E_2 - E_1 \approx (\Gamma_2 - \Gamma_1)\, E_1 \approx 1.091 \times 10^{-16} \times mc^2$ $mc^2 \approx 8.99 \times 10^{16}$ J for 1 kg mass

$E_2 - E_1 \approx 9.8$ J difference in normalized energy of a 1 kg mass elevated 1 m

Therefore, when we use normalized units (coordinate time and proper length), we find that the internal energy of a mass changes with elevation by exactly the amount of gravitational potential energy. For example, a one kilogram mass has 9.8 Joules more energy when it is at an elevation

1 meter above sea level than it has when it is at an elevation of sea level when energy is expressed in normalized units of energy. There is also a change in the normalized speed of light and in the normalized mass. The combination of these factors results in a change in the normalized energy of a mass that is elevated or lowered in a gravitational field. The relationship between internal energy in zero gravity E_o, the normalized internal energy in gravity E_g and gravitational potential energy E_φ is:

$$E_o = E_g - E_\varphi$$

The minus sign in front of E_φ is the result of considering potential energy to be a negative number. This equation leads to the equation $E_o = E_g + E_k$ discussed above. A body in free fall does not change its normalized energy. Previously, a star and planet in an elliptical orbit were used in an example. Sensing the gravity of the star/planet combination by the use of a distant probe mass is equivalent to sensing the zero gravity energy of the star/planet combination.

Mass Transformation: The normalized mass transformation $M_o = M_g/\Gamma$ looks counter intuitive because it indicates that normalized mass increases when gravitational gamma Γ increases. There is no additional matter being created, there is just a change in the perceived inertia when we convert to a single rate of time and quantify units of inertia in locations with different values of Γ. The inertia (mass) exhibited by a body is defined by the force generated when a body is accelerated. Both force and acceleration incorporate units of time therefore mass (inertia) also exhibits a dependence on the rate of time. A combination of factors results in a unit of mass (inertia) in gravity being larger than the same unit of mass (inertia) in zero gravity

Even though normalized mass increases in gravity, normalized energy decreases as explained above. Since it is easier to conceptually understand energy decreasing in gravity, perhaps it is easier to imagine energy being more fundamental than mass (inertia). It is only a historical accident that we use mass as one of the 5 dimensional units of physics. In particle physics, energy is considered more fundamental than mass and units of eV or MeV are common. If energy replaced mass as one of the 5 dimensional units, then the energy transformation would be the single factor that offsets the gravitational effect on time.

The weak equivalence principle says that there is no difference between gravitational mass and inertial mass. This is true because they both scale proportionately to the gravitational gamma when a constant rate of time is used. Merely elevating a mass in the earth's gravitational field changes both the normalized gravitational mass and the normalized inertial mass. The gravitational field produced by the one kilogram mass scales with total energy, so elevating this mass increases the energy of the elevated mass. This energy increase exactly offsets the decrease in energy of whatever means was used to elevate the mass. The total normalized energy and total gravitational field of the earth is unchanged.

Gravitational Redshift: The gravitational red/blue shift is often misinterpreted[1 2]. Above it was shown that the internal energy of an atom changes with elevation by exactly the difference in the gravitational potential energy. Not only is there a change in the internal energy of an atom when it changes elevation, there is also a proportional change in the energy of the atom's energy levels when there is a change in elevation. Therefore, from the perspective of someone in zero gravity (normalized time), a particular atomic transition in gravity is less energetic and this transition emits a comparatively low energy and low frequency photon. This low energy is not detectable locally because all energy comparisons (such as 1 ev) have been similarly shifted to a lower energy by the gravitational effect on time and mass.

Now we will address the gravitational redshift. The formula for the red/blue shift is:

$\lambda_o = \Gamma \lambda_g$ where:
$\lambda_g =$ wavelength in gravity and $\lambda_o =$ wavelength in zero gravity

Suppose a photon in gravity starts at a lower elevation (level 1) and ends at a higher elevation (level 2). If the photon's energy is measured at level 1 and 2 with local instruments, then a loss of energy is observed at level 2. This redshift appears to be a decrease in frequency, a decrease in energy and an increase in wavelength. If it was possible to measure wavelength, frequency and energy from a single elevation (single rate of time), then it would appear as if there was no change in energy, no change in frequency, but the same increase in wavelength that was observed with a local measurement. The energy and frequency disagreement occurs because different rate of time and energy scales are being used at different elevations. The agreement in wavelength occurs because the transformation $L_o = L_g$ says that all observers are using the same length standard to measure wavelength.

If we look only at wavelength, then there is such a thing as gravitational redshift. This is because from all gravitational potentials, the same change in wavelength is observed. However, if we look at either energy or frequency using zero gravity rate of time, then there is no such thing as gravitational redshift. The apparent change in frequency and energy occurs because we measure energy and frequency using local standards of the beginning and ending elevations. At these different elevations (different values of Γ), it is our local standards of energy and time that have changed, not the photon's energy and frequency.

There is an interesting question that I would like to propose. We know that there appears to be a gravitational blue shift when a photon is generated at a high elevation (height 2) and is analyzed at a lower elevation (height 1). In other words, the photon seems to have gained energy. The question is: What would happen if we trapped a photon in a reflecting box at height

[1] L. B. Okun, K.G. Selivanov and V.L. Telegdi," On the Interpretation of the Redshift in a Static Gravitational Field", Am.J.Phys.68:115,2000; arXiv:physics/9907017.
[2] L. B. Okun, "Photons and static gravity, Mod.Phys",Lett.A15:1941,2000; arXiv:hep-ph/0010120

2, then lowered the box to height 1? Would the photon in a box have the same energy as a freely propagating photon when it reached height 1?

I contend that the photon lowered in a box would appear to have less energy than the freely propagating photon. From a local perspective, the freely propagating photon appears to gain energy (blue shifted) and the photon in a box would appear to have the same energy as the energy at height 1. From the perspective of a zero gravity observer, the freely propagating photon retained its original energy when it propagated from heights 2 to 1 and the photon in a box lost energy. This lost energy was removed from the photon in the lowering process. As previously explained, a confined photon exhibits weight. Lowering a box containing a confined photon transfers energy from the photon to the apparatus used to lower the box. There are numerous ways to analyze this problem and I content that they all give the answer described here. However, this is a little off the subject, so I will not elaborate further.

Another question is: How is it possible for the wavelength to change with elevation if there is no change in the normalized frequency? The answer is that the normalized speed of light changes with elevation ($C_o = \Gamma C_g$). If a photon propagates from a lower elevation to a higher elevation, there is no change in frequency, but the normalized speed of light C_g increases with elevation. This increase in the normalized speed of light increases the distance traveled per cycle time (increases the wavelength). This change in wavelength is obvious from any location, but the constant frequency is only observable when all measurements are made from a single elevation (a single rate of time).

Finally, a word of caution about not using the redshift formula ($\lambda_o = \Gamma \lambda_g$) in transformations as a substitute for length. It is not correct to equate wavelength with the unit of length when there is a change in Γ. A photon generated from a local atom has a wavelength that is a good standard of length. However, a photon generated at another gravitational potential has a wavelength that changes with Γ. Therefore, a photon that is not generated locally cannot be used as a standard of length.

Electrical Charge Transformation: The transformation of electric charge needs special explanation. The transformations of $Q_o = Q_g$ is only correct if charge is expressed in coulombs rather than stat coulombs. Coulombs and stat coulombs are fundamentally different. The MKS unit of coulomb is 6.24×10^{18} electrons but the CGS unit of stat coulomb is related to electrostatic force and has units of $\sqrt{ML^3}/T$. The result is that charge is conserved in gravitational transformations if charge is expressed as a number of electrons (coulombs), but it is not conserved if charge is expressed in stat coulombs with units of $\sqrt{ML^3}/T$. If an electrostatic force equation is written in CGS units, then ε_o is missing and there is no normalization of permittivity. The following transformation must be used for charge expressed in stat coulombs:

$Q_o = \sqrt{\Gamma}\, Q_g$ (charge in stat coulombs)

Testing: The objective of these transformations is to offset the gravitational effect on the rate of time and keep the laws of physics covariant in any gravitational potential. It is interesting to make substitutions into various equations of physics and see that the transformations do indeed keep the same equations of physics when there is a transformation of gravitational gamma. This is saying that these transformations exhibit internal self consistency. However, it is also possible to see the implied physics behind these transformations upon close examination.

Shapiro Revisited: In the last chapter we talked about the Shapiro experiment detecting a relativistic increase in the time required for radar to travel radially in the sun's gravity. The delay was equivalent to about a 50 μs delay in the time required for light to travel one way from the earth to the sun's surface. If we use Schwarzschild coordinates, then the 50 μs delay is due entirely to the slowing of the coordinate speed of light which scales according to the local value of Γ^2 along the radial path between the earth and the sun ($c = \Gamma^2 \mathcal{C}_R = \Gamma^2 dR/dt$).

How do we interpret the 50 μs delay using the normalized coordinates? If a tape measure could be used to measure the distance between the earth and the surface of the sun, the distance measured by a tape measured would be about 7.5 km longer than the circumferential radius distance calculated by dividing the circumference by 2π. The normalized coordinate system uses proper length (tape measure length) as coordinate length. Therefore, the normalized coordinates gives the radar pulse credit for having traveled the additional 7.5 km of non-Euclidian distance between the earth and the sun. It takes about 25 μs for speed of light travel to cover 7.5 km, so the normalized coordinates attributes half the 50 μs delay to the time required to travel the non-Euclidian distance generated by the sun's gravity. The other half is due to the normalized speed of light being slowed according to $c = \Gamma C_g = dL_R/dt$. Integrating over the changing Γ along the optical path gives the additional 25 μs due to this slowing. Therefore, the 50 μs total delay is the same, but the interpretation is different.

The following chapters will primarily use the standard definition for the speed of light. This standard definition will be designated by lower case "*c*". Occasionally we will switch into using normalized speed of light to give another perspective. In this case we will use an upper case "*C*". Attention will be called to this change.

Appendix B

This appendix gives the details of the derivation of some of the additional transformations enumerated in the table titled "Gravitational Transformations of Units and Constants". This appendix can be skipped without the loss of any important information to the main points of this book if this backup information is not of interest to the reader.

Velocity v: $v_o \rightarrow \frac{L_o}{T_o} = L_g \frac{\Gamma}{T_g} \rightarrow \Gamma v_g$

$v_o = \Gamma\, v_g$ normalized velocity v_g decreases in gravity (just like C)

Gravitational Constant G: $G_o \rightarrow \frac{L_o^3}{M_o T_o^2} = \frac{L_g^3}{\left(\frac{M_g}{\Gamma}\right)\left(\frac{T_g^2}{\Gamma^2}\right)} \rightarrow \Gamma^3 G_g$

$G_o = \Gamma^3 G_g$ normalized gravitational constant (see comment below)

Energy Density U: $U_o \rightarrow \frac{M_o}{L_o T_o^2} = \frac{\frac{M_g}{\Gamma}}{L_g\left(\frac{T_g^2}{\Gamma^2}\right)} \rightarrow \Gamma U_g$

$U_o = \Gamma U_g$ normalized energy density

Electrical Charge: Next we come to transformations that have dimensions that include electrical charge in Coulombs. These include permittivity ε_o, permeability μ_o, current I, Voltage V and the impedance of free space Z_o. For this exercise, the symbol ε_{oo} will represent ε_o in zero gravity and ε_{og} will represent ε_o in gravity. Similarly we will use the symbols μ_{oo}, μ_{og}, Z_{oo}, and Z_{og}. A unit of charge will be represented by Q_o and Q_g.

It is not possible to use the above substitutions to determine the transformation of a unit of electrical charge when comparing change between zero gravity Q_o and a unit of charge in gravity Q_g using a single rate of time. It is true that the dimension of charge (expressed in Coulombs) does not contain either time or mass, so the two dimensions known to have Γ dependence are missing (stat Coulombs will be discussed later). So superficially it seems as if there should be no change in a unit of charge when the rate of time changes due to a change in Γ. However, we need to supplement this with some additional physical reasoning using the laws of physics.

From the conservation of charge and the Faraday law we know that charge is conserved when there is a change in elevation. This indicates that $Q_o = Q_g$. There is additional support for this contention because the impedance of free space Z_o has units that scale with $1/Q^2$ (dimensional analysis symbol Q). If there was a gravitational dependence on charge, then the impedance of free space would have a gravitational dependence. There would be a slight impedance mismatch when light changes elevation in gravity. This impedance mismatch would cause scattering of electromagnetic radiation from gravitational fields. For all these reasons we will assume that:

$Q_o = Q_g$

A unit of electrical charge in gravity (Coulombs) equals a unit of electrical charge in zero gravity. We will now use this transformation to generate additional electrical transformations.

Permittivity ε_0: $\varepsilon_{oo} \to \dfrac{Q_o^2 T_o^2}{L_o^3 M_o} = \dfrac{Q_g^2\left(\dfrac{T_g^2}{\Gamma^2}\right)}{L_g^3\left(\dfrac{M_g}{\Gamma}\right)} \to \varepsilon_{og}/\Gamma$

$\varepsilon_{oo} = \varepsilon_{og}/\Gamma$ normalized permittivity

Impedance of Free Space Z_0: $Z_{oo} \to \dfrac{M_o L_o^2}{T_o Q_o^2} = \dfrac{\left(\dfrac{M_g}{\Gamma}\right)L_g^2}{\left(\dfrac{T_g}{\Gamma}\right)Q_g^2} \to Z_{og}$

$Z_{oo} = Z_{og}$ impedance of free space

Voltage V: $V_o \to \dfrac{M_o L_o^2}{T_o^2 Q_o} = \dfrac{\left(\dfrac{M_g}{\Gamma}\right)L_g^2}{\left(\dfrac{T_g^2}{\Gamma^2}\right)Q_g} \to \Gamma V_g$

Temperature Θ: Finally we come to the transformation of Boltzmann's constant k and temperature. Boltzmann's constant is typically described as: $k = 1.38\ 10^{-23}$ Joule/molecule ^{0}Kelvin. This number, measured locally, does not change when gravitational gamma is changed. However, the standard of what constitutes a unit of energy (Joule) changes according to the previously derived transformation: $E_g = E_o/\Gamma$. Therefore, to an observer using normalized time, the energy per molecule per degree Kelvin decreases in gravity. Therefore, an acceptable interpretation for the zero gravity observer is that temperature in gravity equals the same temperature in zero gravity, but the Boltzmann "constant" depends on Γ.
$\Theta_o = \Theta_g$ temperature is unaffected by an change in gravity

Boltzmann Constant: $k_B \to ML^2/T^2\Theta$

$k_B = 1.38\ 10^{-23}$ Joule/molecule ^{0}Kelvin → $ML^2/T^2\Theta$ molecule

$k_{Bo} \to \dfrac{M_o L_o^2}{T_o^2 \Theta_o}$ molecule $= \dfrac{\left(\dfrac{M_g}{\Gamma}\right)L_g^2}{\left(\dfrac{T_g^2}{\Gamma^2}\right)\Theta_g}$ molecule $\to \Gamma k_{Bg}$

$k_{Bo} = \Gamma k_{Bg}$ normalized Boltzmann's "constant"

Stefan-Boltzmann Constant: $\sigma_o \to \frac{M_o}{T_o^3 \Theta_o^4} = \frac{\frac{M_g}{\Gamma}}{\left(\frac{T_g^3}{\Gamma^3}\right)\Theta_g^4} \to \sigma_g / \Gamma^2$

$\sigma_o = \Gamma^2 \sigma_g$ normalized Stefan-Boltzmann "constant"

The Stefan-Boltzmann constant is the constant associated with the intensity of a black body emission $\mathcal{I} = \sigma\varepsilon T^4$ where $\sigma =$ Stefan-Boltzmann Constant, $\varepsilon =$ emissivity, T is temperature and Θ is the dimension of temperature. The equation $\mathcal{I} = \sigma\varepsilon T^4$ supports the idea that $\Theta_g = \Theta_o$ because temperature is raised to the fourth power. If there was a temperature dependence on Γ, we would have a Γ^4 dependence.